One Day Workshop on Applied Mathematics

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Abstracts

Nonlinear PDEs for computer vision and 3D-reconstruction

Maurizio Falcone
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Abstract. 3D-reconstruction techniques aim at inferring the 3D geometry of a scene, given one or several 2D images of this scene. They do so by analyzing the luminous quantities in the images. A physics-based model for the observed colors is introduced, which describes the interactions between the geometry, the reflectance of the observed surface, the lighting and the camera. Then, this model needs to be inverted in each pixel in order to recover the geometry. In this talk, we describe some recent advances regarding modeling and resolution of shape-from-shading (single image) and photometric stereo (multiple images obtained under varying lighting). We show that resorting to PDEs is a natural way to cope with some of the arising ambiguities, but also to elegantly formulate the problem by reducing the number of unknowns and priors. Nevertheless, the resulting PDEs are usually nonlinear and cannot be solved exactly also because of measurement noise. Hence, we discuss possible numerical approaches to solve these problems.

An optimal junction solver for traffic flow

Mauro Garavello
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Abstract. We consider a junction composed by m incoming roads (modeled by the real interval $I_i = (-\infty, 0)$) and n outgoing ones (modeled by the real interval $I_i = (0, +\infty)$). On each road I_i we consider the Lighthill-Whitham-Richards model for traffic

$$\partial_t u_i + \partial_x f(u_i) = 0 \tag{1}$$

where $u_i = u_i(t, x)$ denotes the density of traffic at time t and position x in the i-th road I_i and $f : \mathbb{R} \to \mathbb{R}$ is the flux. Given an initial datum $\bar{u}_i$ on each road, we consider the Cauchy problem at the junction

$$(2) \qquad \begin{cases} \partial_t u_1 + \partial_x f(u_1) = 0 & x \in I_1,\ t > 0 \\ \quad \vdots & \\ \partial_t u_{n+m} + \partial_x f(u_{n+m}) = 0 & x \in I_{n+m},\ t > 0 \\ u_1(0, x) = \bar{u}_1(x) & x \in I_1 \\ \quad \vdots & \\ u_{n+m}(0, x) = \bar{u}_{n+m}(x) & x \in I_{n+m}. \end{cases}$$

We address our attention to an optimal control problem for system (2). More precisely, given $T > 0$ and a function $\mathcal{J} : \mathbb{R}^m \to \mathbb{R}$, we want to select a solution to (2) which maximizes the functional

$$(3) \qquad \int_0^T \mathcal{J}\left(f(u_1(t, 0)), \ldots, f(u_m(t, 0))\right) dt$$

and which respects additional constraints describing the preferences of drivers. The functional (3) depends on the traces of the fluxes of the solution to (2) at the junction.

We show that, under suitable assumptions, this maximization problem admits solutions. In general there are infinitely many solutions to the Cauchy problem (2), which maximize (3). Moreover we present some additional criteria in order to select an optimal solution.

Joint work with F. Ancona, A. Cesaroni, and G. M. Coclite.

Graph partitioning using differential equations

Nicola Guglielmi
University of L'Aquila

Abstract. For a given weighted undirected graph, we study two peculiar aspects: the minimum cut problem and the clustering problem. For the first problem, which is solved very efficiently by discrete optimization methods, we consider some additional constraints, like membership and cardinality constraints. In such situations the problem appear to be NP-hard. Clustering of a graph may be efficiently done by the so-called Fiedler vector, the eigenvector associated to the second smallest eigenvalue of the Laplacian of the weight matrix corresponding to the graph. However, the information might be ambiguous if there is a small perturbation determining the coalescence of the second and third eigenvalues.

In this talk we propose a methodology - based on the numerical solution of a system of differential equations - to deal with both problems.

Examples will be given to illustrate the behavior of the method.

This is a joint work with E. Andreotti (L'Aquila) and D. Edelmann and C. Lubich (Tuebingen).

Titolo

Pierangelo Marcati
University of L'Aquila

Abstract.......

Continuum limit for metamaterials: an inverse problem

Mario Pulvirenti
University of L'Aquila

Abstract. We consider a quite general one dimensional continuum. Our aim is to construct and analyze discrete approximations in terms of physically realizable mechanical systems. We validate our construction by proving a convergence theorem of the microscopic system to the given continuum, as the scale parameter goes to zero. In other words we are interested on an inverse problem: given some macroscopic properties, find the microscopic mechanical system which realizes it. This is a joint work with A. Carcaterra, F. dell'Isola and R. Esposito.

Titolo

Marco Sammartino
University of Palermo

Abstract.......